**Homework 04.**

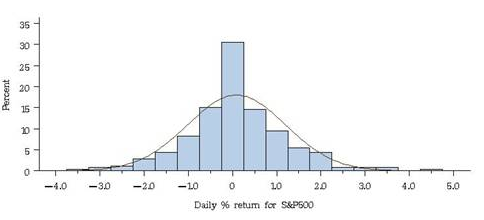
In this assignment you will have a chance to see how probability is used in the real-world.

**Assigned: 5 September 2017**

**Due: not to be turned in**

**S&P500.** As reported by Bloomberg (see <http://www.bloomberg.com/news/2012-09-03/earnings-matter-most-for-u-s-stocks-as-economic-obsession-fades.html)>, “The so-called correlation coefficient among S&P 500 companies fell to 0.58 on Aug. 31 … Correlation reached a record 0.86 percent in October.”

From finance.yahoo.com, the average daily return for the stock market last year (from 9/26/2011 to 9/26/2012) had an average daily return of 0.089%, and a standard deviation of 1.111%, and the returns were close to normally distributed:



Suppose that every stock in the S&P500 has a daily return that is normally distributed with an expected daily return of 0.089% and a standard error of 1.111%.

1. What is the chance that, for a randomly selected day, that a randomly-chosen stock in the S&P500 has a return greater than 1.5%? Choose the answer that is closest to correct.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1. 2% | 1. 10% | 1. 35% | 1. 13% | 1. 0.00% |

Returns for stocks are normally distributed with mean of 0.089 and SE = 1.111.

P(return > 1.5) = P(Z > (1.5 – 0.089) / 1.111) = P(Z > 1.27) = 0.1020

1. Consider a portfolio that gives equal weight to the two stocks.  Assume that, for each stock, the daily returns have an expected value of 0.089% and a standard error of 1.111%, and that the correlation between daily returns is 0.58. What is the chance that the two-stock loses money on a given day? Choose the answer that is closest to correct.

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| --- | --- | --- | --- | --- |
| 1. 9% | 1. 11% | 1. 43% | D) 46% | E) 28% |

Let X be the return for stock #1, and let Y be the return for stock #2.

E(X) = E(Y) = 0.089, SE(X) = SE(Y) = 1.111; corr(X,Y) = 0.58 🡪 cov(X,Y) = 0.7159

Define R = 0.5X + 0.5Y [equal weight to two stocks]

E(R) = 0.089, SE(R) = sqrt[a2SE(X)2 + 2 a b cov(X,Y) + b2SE(Y)2]

= sqrt[(0.5)2 (1.111)2 + 2(0.5)(0.5)(0.7159) + (0.5)2 (1.111)2] = 0.9875

P(R < 0) = P(Z < (0 – 0.089) / 0.9875) = P(Z < –0.09) = 0.4641

1. Consider a portfolio that gives equal weight to the two stocks.  Assume that, for each stock, the daily returns have an expected value of 0.089% and a standard error of 1.111%, but assume now that the correlation between daily returns is 0.86. Now what is the chance that this two-stock portfolio loses money on a given day? Choose the answer that is closest to correct.

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| --- | --- | --- | --- | --- |
| 1. 8% | 1. 27% | 1. 38% | 1. 47% | 1. 54% |

As above, but cov(X,Y) = 1.0615, SE(R) = 1.0714

P(R < 0) = P(Z < (0 – 0.089) / 1.0714) = P(Z < –0.083) = 0.4669

1. Consider a portfolio that is equally divided among 500 stocks; assume that, for each stock, the daily returns have an expected value of 0.089% and a standard error of 1.111%. If all stocks were uncorrelated, what is the chance that the 500-stock portfolio loses money on a given day? Choose the answer that is closest to correct.

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| --- | --- | --- | --- | --- |
| 1. 2% | 1. 4% | 1. 0.3% | 1. 12% | 1. 46% |

The return is the average of 500 independent stocks, all with the same distribution…

E(avg) = 0.089, SE(avg) = 1.111 / sqrt(500) = 0.04969

P(avg < 0) = P(Z < (0 – 0.089) / 0.04969) = P(Z < –1.79) = 0.0366

1. Consider a portfolio that is equally divided among 500 stocks; assume that, for each stock, the daily returns have an expected value of 0.089% and a standard error of 1.111%, and that the correlation between any two stocks is 0.58; i.e., the correlation matrix would be

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | X1 | X2 | X3 | ... | X499 | X500 |
| X1 | 1.00 | 0.58 | 0.58 | … | 0.58 | 0.58 |
| X2 | 0.58 | 1.00 | 0.58 |  | 0.58 | 0.58 |
| X3 | 0.58 | 0.58 | 1.00 | … | 0.58 | 0.58 |
| … | … | … | … | … | … | … |
| X499 | 0.58 | 0.58 | 0.58 | … | 1.00 | 0.58 |
| X500 | 0.58 | 0.58 | 0.58 | … | 0.58 | 1.00 |

What is the chance that the 500-stock portfolio loses money on a given day?  [Hint:  how many covariance terms are there when there are 500 stocks?]

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1. 46% | 1. 2% | 1. 4% | 1. 0.3% | 1. 12% |

Hint: given 500 stocks, there are 500 SE’s, and 5002 – 500 = 249,500 covariances

Define R = (1/500)X1 + (1/500)X2 + … + (1/500)X500

E(R) = 0.089,

SE(R) = sqrt[500\*(1/500)2(1.111)2 + 249500\*(1/500)\*(1/500)(0.7159)] = 0.8467

P(R < 0) = P(Z < (0 – 0.089) / 0.8467) = P(Z < –0.105) = 0.458

**Fiscal Cliff.** In September 2012, Moody’s Analytics published an analysis of the US economy (see <http://www.marketwatch.com/story/moodys-analytics-adds-us-fiscal-cliff-scenarios-to-its-forecast-database-2012-09-18>). According to the analysis, there were three possible scenarios:

* “Cliff” (15% chance), where GDP = –2.8% and unemployment will be 9%
* “No cliff” (30% chance), where GDP = +2.9% and unemployment will be 6%
* “Most likely” (55% chance), where GDP = 0.0% and unemployment will be 8%

1. If an analyst wanted to make the correlation between GDP and unemployment –1.0, what would GDP have to be in the “most likely” case (assuming the rest of the numbers are unchanged)?

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| --- | --- | --- | --- | --- |
| 1. -1.9% | 1. -0.9% | 1. 0.0% | 1. 0.9% | 1. 1.9% |

A correlation of -1.0 means a perfectly linear relationship between GDP and unemployment:

Unemployment 6.0 7.0 8.0 9.0

Change in GDP 2.9 ? ? -2.8

For unemployment, 8.0 is one third of the distance between 9.0 and 6.0; so, for a perfectly linear relationship, the GDP for an unemployment of 8.0 needs to be one third of the distance between –2.8 and +2.9. The distance between –2.8 and 2.9 is 5.7; a third of the distance is 5.7 / 3 = 1.9; and so “change in GDP” must be -2.8 + 1.9 = –0.9.

(The answer is easiest to see by doing a plot)

**Apple (AAPL).** Data are available from <http://finance.yahoo.com> for 7310 days of the daily change in Apple (AAPL) stock (9/7/1984 to 9/13/2013). The value of AAPL shares went up 49.2% of the time and declined 50.8% of the time. The daily percentage returns had an average of 0.116% and a standard deviation of 2.985%, and the daily returns were independent of each other. There are 252 of trading days per year.

Suppose an investor begins each day with $1000 worth of AAPL, and at the end of the day she buys or sells enough shares of AAPL that she begins the next day with $1000 worth of AAPL.

1. If returns for AAPL were normally distributed, what percentage of days should AAPL have had a positive return, if daily returns had an average of 0.116% and a standard deviation of 2.985%?

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1. 14% | 1. 38% | 1. 48% | D) 52% | E) 96% |

Given a normal distribution with  = 0.116 and  = 2.985, the chance that the value > 0 is equivalent to P(Z > –0.0388) = 0.5155, or **51.55%**

1. Suppose the investor performs her “start each day with $1000 worth of AAPL” strategy for two years. How likely is it that she will have made money on at least half of those 504 days? Express your answer as a percentage.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1. 11% | 1. 26% | C) 74% | D) 77% | E) 81% |

The chance of a positive return per day is 0.5155. Given 504 (independent) days, the expected number of positive days is E(count) = np = 259.81 with SE(count) = sqrt( np(1–p)) = 11.22. The chance of making money on at least half the days (252 or more days) is P(count > 251.5) = P(Z > –0.74) = 0.7707, or **77.06%**

1. Suppose the investor does this strategy for two years (i.e., starting each day with $1000 worth of AAPL). How likely is it that she will have made money at the end of the 504 days? Express your answer as a percentage. (Ignore transaction costs.)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1. 16% | 1. 29% | 1. 58% | 1. 67% | 1. 81% |

With $1000 a day, the expected return per day is 0.00116 \* $1000 = $1.16, with a standard deviation of 0.02985 \* $1000 = $29.85. Given 504 (independent) days, the expected sum is E(sum) = n E(X) = 584.64, with SE = sqrt(n) SE(X) = 670.13. The chance that the sum will be positive is P(sum > 0) = P(Z > –0.872) = 0.8085, or **80.85%**.

**Determining value**. Management at Company X is thinking about buying a smaller company. The value of the smaller company depends on what Congress decides about the “fiscal cliff”. According to Goldman-Sachs, there is a 30% chance that Congress does nothing and the U.S. goes off the “fiscal cliff”; if the U.S. goes off the “fiscal cliff”, the smaller company would be worth $50M in 2013. If Congress reaches a “compromise” (assume this has a 60% chance), the smaller company would be worth $100M in 2013. Finally, if the U.S. arrives at a “solution”, the company would be worth $150M in 2013. The smaller company is currently being sold for is being sold for $95M.

Company X is not currently interested in buying the smaller company; by their calculations, the smaller company is only worth $90M. However, the company can pay a fee to “lock in” the purchase price of $95M in order to wait to see what Congress does. After seeing what Congress does, Management at Company X can then decide whether to spend $95M in 2013 to purchase the smaller company. (The fee is nonrefundable, regardless of whether Management spends the additional $95M.)

1. Under what conditions does it make sense for Company X to pay the fee? (You may ignore inflation and the “time value of money”.)
   1. It makes financial sense if the fee is less than $8.5M and does not make financial sense if the fee is more than $8.5M
   2. It makes financial sense if the fee is less than $12M and does not make financial sense if the fee is more than $12M
   3. It makes financial sense if the fee is less than $5M and does not make financial sense if the fee is more than $5M
   4. It makes financial sense if the fee is less than $5.5M and does not make financial sense if the fee is more than $5.5M
   5. Since the smaller company currently has an expected value of $90M, it would never make sense to pay the fee, since it would never make sense to pay $95M for the smaller company.

Depending on what Congress does, there is a 30% chance the company is worth zero as an acquisition [don’t spend $95M for a $50M company!], a 60% chance the company is worth $5M [$100M value, less $95M cost], and a 10% chance the company is worth $55M [$150M value, less $95M cost]. Thus, waiting to see what Congress does is worth (0)(0.30) + (5)(0.60) + (55)(0.1) = $8.5M